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Mathematical model for changing the surface roughness of aluminum alloys when changing the final treatment modes

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Abstract. In the process of multivariate design of machining technology, one of the important tasks is the selection of optimal cutting conditions at the final transitions of the shaping process, ensuring a given surface quality on a workpiece made of the corresponding material. At the same time, to describe the majority of design procedures for the machining process, there are problems associated with the algorithm for choosing a solution method, response function, and areas of feasible solutions at all stages of processing. In this paper, as an algorithm for solving the problem, it is proposed to use the well-known technique of extreme experimental design, which allows to obtain a mathematical model of the investigated multifactor process with incomplete knowledge of its optimization mechanism. Such a study allows us to simplify the procedure for setting the processing regime of the workpiece at the final stage in the conditions of the existing production, while ensuring the specified quality of the machined surfaces and maintaining the productivity of the forming process. This technique is the subject of consideration in this work.

1. Introduction

One of the important tasks facing the machine-building enterprises is to ensure high quality of manufactured products while maintaining the required productivity of the machining process.

First of all, this concerns the provision of specified qualitative indicators of the processing process at the finishing and finishing stages, where the required height of microroughnesses of the surfaces is directly determined by the choice of rational processing modes in the form of cutting depth t , tool feed s and cutting speed v . These factors, on the other hand, uniquely determine the productivity of the cutting process itself [1].

When processing non-ferrous metals with various types of milling cutters, in some cases it is necessary to prescribe a priori cutting conditions that allow achieving the specified quality of the finished surfaces, which undoubtedly immediately begins to affect the decrease in the productivity parameter of the processing process.

The solution to the existing problem is seen in conducting experimental engineering studies for the subsequent construction of mathematical models that allow us to describe the optimal conditions for the course of the machining process, which is especially important at its final stages.

In this paper, we study the process of the influence of the cutting mode on the height of microroughness when milling flat surfaces of body parts at the final processing stage, as well as the search for conditions that ensure the optimum surface quality specified by the working drawing using the experimental extreme planning.



2. Initial data of the experiment

To conduct a multifactor experiment, the initial workpiece was used in the form of a plate of AMg6 alloy GOST 4784-97. The study was conducted on a 675 model of a universal milling machine. An end mill $d = 25$ mm made of P6M5 high-speed steel with the following geometry was used as a cutting tool:

- 1) main angle in plane $\varphi = 60^\circ$;
- 2) angle of inclination of the cutting edge $\lambda = 5^\circ$;
- 3) rake angle $\gamma = 10^\circ$.

At the initial stage, it was necessary to determine the response function for the subsequent determination of optimal conditions for the processing process [2]. This dependence should meet, first of all, the requirement of reproducibility and controllability [3]. In accordance with similar requirements, the object of study was presented in the form of surface roughness Ra , and the response function, which is also an optimization parameter, was formulated as the following expression:

$$Ra = f(x_1, x_2, x_3), \quad (1)$$

where x_1 is the first factor in the form of cutting speed, m/min;
 x_2 is the second factor in the form of a mill feed, mm/rev;
 x_3 is the third factor in the form of cutting depth, mm.

Since dependence (1) is unknown on three variable factors, the expression of the response function must be presented in the form of a polynomial of the first degree, which takes into account linear effects and interaction effects [3], [4], [5]. For three factors, this polynomial will have the following form:

$$y = b_0 + b_1x_1 + b_2x_2 + b_3x_3 + b_{12}x_1x_2 + b_{13}x_1x_3 + b_{23}x_2x_3, \quad (2)$$

where $b_0, b_1, b_2, b_3, b_{12}, b_{13}, b_{23}$ are regression coefficients;
 y is a sample estimate of the response function Ra .

All factors that are presented in dependence (1) are quantitative.

The levels and ranges of variation of factors that were adopted for the experimental study are given in table 1.

The coded values of the factors x_1, x_2, x_3 are taken equal to plus one at the upper level, zero at the basic level and minus one at the lower level for the natural values of these factors (table 1).

Table 1. Levels and intervals of factor variation.

Factors	Designation	The range of variation	Factor level		
			Upper +1	Main 0	Bottom -1
Cutting speed, (m/min)	x_1	+25 -23	130	105	82
Mill feed, (mm/rev)	x_2	0,05	0.2	0.15	0.1
Cutting depth, (mm)	x_3	0,25	1	0.75	0.5

Factor levels and variation intervals were selected based on the results of a preliminary experiment, being guided by data from the reference literature and equipment capabilities in the mode of final milling of the non-ferrous metal part plane [6], [7].

3. A full factorial experiment

To determine the regression coefficients in dependence (2), a complete factorial experiment of type 2^3 was planned, where parameter 3 represents the number of factors in dependence (1), and parameter 2 determines the variation of these factors at two levels [8], [9].

The planning matrix of the full factorial experiment 2^3 and the results of measuring the height of the microroughness are presented in table 2.

At the same time, the surface roughness values Ra in microns were measured in all eight experiments using a *Surftest SJ-301* profilometer.

Table 2. Matrix of a complete factor experiment type 2^3 and results of experiments.

№ experience's	X_0	X_1	X_2	X_3	$X_1 X_2$	$X_1 X_3$	$X_2 X_3$	y (mkm)
1	+	−	−	−	+	+	+	2.2
2	+	+	−	−	−	−	+	1.98
3	+	−	+	−	−	+	−	2.6
4	+	+	+	−	+	−	−	3.38
5	+	−	−	+	+	−	−	2.25
6	+	+	−	+	−	+	−	2.15
7	+	−	+	+	−	−	+	3.07
8	+	+	+	+	+	+	+	3.5

In accordance with the results of an experimental study, the regression coefficients for the dependencies were determined, which take into account the value of the free term b_0 , linear effects b_1 , b_2 , b_3 and interaction effects b_{12} , b_{13} , b_{23} in equation (2) [8], [9]. The dependencies that determine these coefficients and their final values are presented in the following expressions:

$$b_0 = \frac{\sum_{j=1}^N y_j}{N} = \frac{2.2 + 1.98 + 2.6 + 3.38 + 2.25 + 2.15 + 3.07 + 3.5}{8} = 2.64. \quad (3)$$

$$b_1 = \frac{\sum_{j=1}^N x_{1j} y_j}{N} = \frac{-2.2 + 1.98 - 2.6 + 3.38 - 2.25 + 2.15 - 3.07 + 3.5}{8} = 0.11. \quad (4)$$

$$b_2 = \frac{\sum_{j=1}^N x_{2j} y_j}{N} = \frac{-2.2 - 1.98 + 2.6 + 3.38 - 2.25 - 2.15 + 3.07 + 3.5}{8} = 0.5. \quad (5)$$

$$b_3 = \frac{\sum_{j=1}^N x_{3j} y_j}{N} = \frac{-2.2 - 1.98 - 2.6 - 3.38 + 2.25 + 2.15 + 3.07 + 3.5}{8} = 0.1. \quad (6)$$

$$b_{12} = \frac{\sum_{j=1}^N x_{1j} x_{2j} y_j}{N} = \frac{2.2 - 1.98 - 2.6 + 3.38 + 2.25 - 2.15 - 3.07 + 3.5}{8} = 0.19. \quad (7)$$

$$b_{13} = \frac{\sum_{j=1}^N x_{1j} x_{3j} y_j}{N} = \frac{2.2 - 1.98 + 2.6 - 3.38 - 2.25 + 2.15 - 3.07 + 3.5}{8} = -0.03. \quad (8)$$

$$b_{23} = \frac{\sum_{j=1}^N x_{2j} x_{3j} y_j}{N} = \frac{2.2 + 1.93 - 2.6 - 3.38 - 2.25 - 2.15 + 3.07 + 3.5}{8} = 0.05. \quad (9)$$

Given the obtained values of the regression coefficients, polynomial (2) will take the following form:

$$y = 2.64 + 0.11x_1 + 0.5x_2 + 0.1x_3 + 0.19x_1x_2 - 0.03x_1x_3 + 0.05x_2x_3. \quad (10)$$

4. Evaluating the adequacy of the mathematical model

Next, the statistical significance of all the coefficients that form the dependence (2) was estimated. For such an assessment, it is sufficient to determine the variance of the optimization parameter by the number of parallel experiments S_y^2 and the adequacy of the mathematical model by the Fisher criterion F [4], [5], [10], [11].

To determine the variance of the optimization parameter, 4 parallel experiments m were carried out with the factors at the basic level (table 1). The values of the optimization parameter Ra obtained in the experiments, its average values of Ra_{av} , deviations from the mean values $(Ra - Ra_{av})$ and the squares of these deviations are given in table 3.

Table 3. Data for calculating the variance of the object under study.

№ experience's	Ra , (mkm)	Ra_{av} , (mkm)	$(Ra - Ra_{av})$	$(Ra - Ra_{av})^2$
1	2.69	$Ra_{av} = \frac{\sum Ra_i}{4} = 2.68$	0.01	0.0001
2	2.76		0.08	0.0064
3	2.6		-0.08	0.0064
4	2.68		0	0
				$\Sigma(Ra - Ra_{av})^2 = 0.0129$

It is known that the variance of the studied parameter Ra can be determined by the following dependence [5], [10], [11]:

$$S_y^2 = \frac{\Sigma(Ra - Ra_{av})^2}{m-1}. \quad (11)$$

Where, in accordance with the data from table 3 and equation (11) it follows that $S_y^2 = \frac{0.0129}{3} = 0.0043$.

Further, for eight independent experiments, the mean square error S_b in determining all the coefficients in the regression equation (2) will be equal to the following value:

$$S_b = \sqrt{\frac{S_y^2}{N}} = \sqrt{\frac{0.0043}{8}} = 0.023. \quad (12)$$

The confidence interval for the coefficients in the regression equation (2) was determined by the following formula [8], [9]:

$$\Delta b = \pm(t_{q,f} \cdot S_b), \quad (13)$$

where $t_{q,f}$ is Student's criterion, which is necessary for sifting out dubious results when determining coefficients in the regression equation.

The value of this criterion at a 5% significance level and the number of degrees of freedom of the model $f = m - 1 = 4 - 1 = 3$ will correspond to 3.18. Therefore, the confidence interval for the coefficients in equation (2) will be $\Delta b = \pm(3.18 \cdot 0.023) = \pm 0.073$.

Obviously, the values of the coefficients b_{13} , b_{23} in dependence (10), which characterize the effects of the interaction, are significantly less than the confidence interval. This means that these coefficients can be neglected due to their insignificant effect on the surface roughness.

Taking into account the numerical data obtained as a result of checking the statistical significance of the coefficients, the regression equation will take the following form:

$$y = 2.64 + 0.11x_1 + 0.5x_2 + 0.1x_3 + 0.19x_1x_2. \quad (14)$$

The second check, which consists in assessing the adequacy of the finally formed mathematical model (14), was carried out according to the Fisher criterion F .

To perform this check, the calculated value of the Fisher criterion F_p was determined, and based on the number of degrees of freedom of the mathematical model, a tabular value of this criterion F_t was chosen at a 5% level of significance.

The condition for the adequacy of the model is the fulfillment of the inequality [5]:

$$F_p \leq F_t. \quad (15)$$

In the first step of this test, the adequacy variance was determined S_{ad}^2 .

To determine it, an auxiliary table 4 was created with the values of the studied parameter, which were obtained during the experiment and as a result of calculation according to dependence (14).

Table 4. Supporting data for calculating the dispersion of adequacy.

№ experience's	Ra_{exp} , (mkm)	Ra_{calc} , (mkm)	$(Ra_{exp} - Ra_{calc})$	$(Ra_{exp} - Ra_{calc})^2$
1	2.2	2.12	0.08	0.0064
2	1.98	1.96	0.02	0.0004
3	2.6	2.74	-0.14	0.0196
4	3.38	3.34	0.04	0.0016
5	2.25	2.32	-0.07	0.0049
6	2.15	2.16	-0.01	0.0001
7	3.07	2.94	0.13	0.0169
8	3.5	3.54	-0.04	0.0016
				$\Sigma(Ra_{exp} - Ra_{calc})^2 = 0.052$

The adequacy variance was determined by the dependence (2):

$$S_{ad}^2 = \frac{\Sigma(Ra_{exp} - Ra_{calc})^2}{f}, \quad (16)$$

where f is the number of degrees of freedom of the mathematical model.

With the number of experiments N equal to eight and the number of factors k in the mathematical model equal to 3, the adequacy variance based on dependence (16) will take the following value:

$$S_{ad}^2 = \frac{0.052}{(N - (k + 1))} = \frac{0.052}{(8 - (3 + 1))} = 0.013.$$

Further, according to the value of the adequacy variance S_{ad}^2 and the variance of the studied parameter S_y^2 , the calculated value of the Fisher criterion was determined by the following formula:

$$F_p = \frac{S_{ad}^2}{S_y^2}. \quad (17)$$

Whence F_p will have the following value $F_p = 0.013/0.0043 = 3$.

The table value of the Fisher criterion with the number of degrees of freedom for the larger variance $f_1 = 4$ and for the smaller variance $f_2 = 3$ takes the following value $F_t = 9.1$ [8], [9].

Consequently, inequality (15) is fulfilled, which means that model (14) can be recognized as adequate.

5. Transition from coded values of factors to their actual values

To obtain the final experimental regression equation in accordance with the type of polynomial (14), it is necessary to carry out the transition from the coded values of factors to their natural values.

A similar transition was made using the following formula [8], [9], [12]:

$$x_i = \frac{X_i - X_i^0}{\varepsilon_i}, \quad (18)$$

where i is the factor number;

ε_i is the interval of variation of the factor;

X_i is the actual value of the i -th factor;

X_i^0 is the actual value of the main level of the factor.

The transition formulas to the natural values of the processing mode were formulated as follows [8], [9], [12]:

$$\text{a) for cutting speed } x_1 = \frac{v - 105}{25}; \quad (19)$$

$$\text{b) for cutter feeding } x_2 = \frac{s - 0,15}{0,05}; \quad (20)$$

$$\text{c) for cutting depth } x_3 = \frac{t - 0,75}{0,25}. \quad (21)$$

After substituting the dependencies (19), (20) and (21) in equation (14), the regression equation was obtained in the final form:

$$Ra = 2.77 - 0.016 \cdot v - 5.69 \cdot s + 0.4 \cdot t + 0.15 \cdot v \cdot s. \quad (22)$$

6. Conclusion

As a result of an experimental study and processed experimental data, the following conclusions can be formulated:

1. From the obtained regression equation (22), it follows that the value of the surface roughness Ra during the final processing of the aluminum alloy is most affected by the feed rate s , and the degree of its influence is greater than the degree of influence of other factors.

2. In the area of the experiment (table 1), with a decrease in tool feed and an increase in cutting speed and depth, dependence (22) revealed a local extremum region with maximum roughness after the 5-th experiment (figure 1).

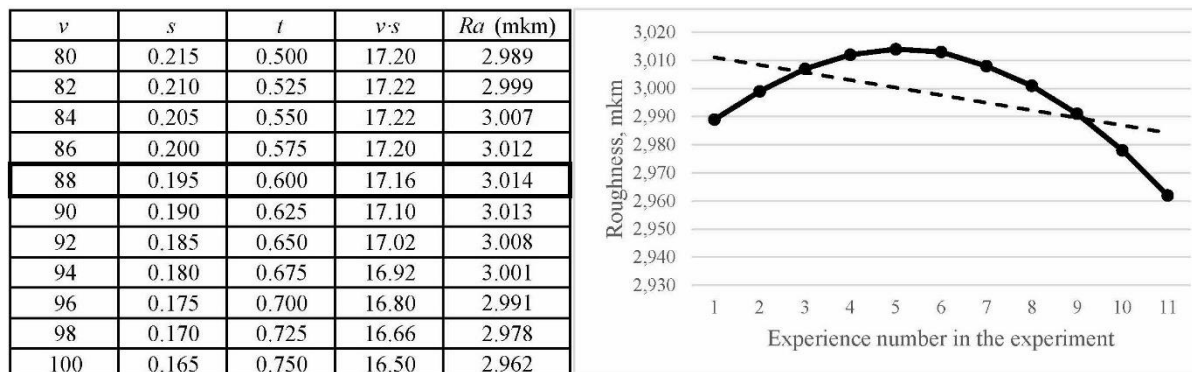


Figure 1. Description of the local extremum region.

3. Dependence (22) can be effectively used to search for local optima during the course of the cutting process on workpieces made of aluminum alloys processed by end mills.

4. The results can be used to select the cutting conditions for flat surfaces on aluminum alloy body blanks at the final processing stages in the real production conditions, which will ensure the roughness specified by the drawing while maintaining the desired performance of the forming process.

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